

$$\int \frac{5x-1}{(x-3)(x+4)} dx = \int \left[\frac{2}{x-3} + \frac{3}{x+4} \right] dx$$

$$2 \ln|x-3| + 3 \ln|x+4| + C$$

$$\frac{5x-1}{(x-3)(x+4)} = \frac{A(x+4)}{(x-3)(x+4)} + \frac{B(x-3)}{(x+4)(x-3)}$$

$$\frac{5x-1}{(x-3)(x+4)} = \frac{Ax+4A}{(x-3)(x+4)} + \frac{Bx-3B}{(x-3)(x+4)}$$

$$Ax+Bx=5x$$

$$A+B=5 \Rightarrow A=2 \quad B=3$$

$$4A-3B=-1$$

$$\frac{(5x-1)}{(x-3)(x+4)} = \frac{Ax+Bx+4A-3B}{(x-3)(x+4)}$$

$$\begin{aligned} (A+B=5) & \Rightarrow A+3B=15 \\ 4A-3B & = -1 \\ \hline 7A & = 14 \\ A & = 2 \end{aligned}$$

$$\frac{6x+7}{(x+2)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2} = \frac{A(x+2)}{(x+2)^2} + \frac{B}{(x+2)^2} = \frac{6}{x+2} - \frac{5}{(x+2)^2}$$

$$\frac{6x+7}{(x+2)^2} = \frac{Ax+2A+B}{(x+2)^2}$$

$$Ax=6x$$

$$A=6$$

$$7=2A+B=2 \cdot 6+B$$

$$-5=B$$

$$\begin{array}{r}
 x^2 - 2x - 3 \overline{) 2x^3 - 4x^2 - x - 3} \\
 \underline{-(2x^3 - 4x^2 - 6x)} \\
 0 + 0 + 5x - 3
 \end{array}$$

$$\frac{2x^3 - 4x^2 - x - 3}{x^2 - 2x - 3} = 2x + \frac{5x - 3}{x^2 - 2x - 3} = 2x + \frac{5x - 3}{(x-3)(x+1)} = 2x + \frac{A}{x-3} + \frac{B}{x+1}$$

$$2x + \frac{A(x+1)}{(x-3)(x+1)} + \frac{B(x-3)}{(x+1)(x-3)}$$

$$2x + \frac{Ax + A}{(x-3)(x+1)} + \frac{Bx - 3B}{(x+1)(x-3)} = 2x + \frac{5x - 3}{(x-3)(x+1)}$$

$$\begin{array}{l}
 Ax + Bx = 5x \\
 A - 3B = -3 \\
 A + B = 5 \Rightarrow B = 2 \\
 A + B = 5 \Rightarrow A = 3 \\
 \hline
 4B = 8 \\
 B = 2
 \end{array}$$

$$\frac{2x^3 - 4x^2 - x - 3}{x^2 - 2x - 3} = 2x + \frac{3}{x-3} + \frac{2}{x+1}$$

$$\frac{x-2}{(x+1)(x-1)^2} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2} = \frac{-\frac{3}{4}}{x+1} + \frac{\frac{3}{4}}{x-1} + \frac{-\frac{1}{2}}{(x-1)^2}$$

$Ax^2 + Bx^2 = 0$
 $A = -B$
 $A = -\frac{3}{4}$
 $B = \frac{3}{4}$

$$\frac{A(x-1)^2}{(x+1)(x-1)^2} + \frac{B(x+1)(x-1)}{(x+1)(x-1)^2} + \frac{C(x+1)}{(x+1)(x-1)^2}$$

$$Ax^2 - 2Ax + A + Bx^2 - B + Cx + C = x - 2$$

$$-2A + C = 1 \Rightarrow C = 1 + 2A$$

$$A - B + C = -2$$

$$A + A + 1 + 2A = -2$$

$$4A = -3 \Rightarrow A = -\frac{3}{4}$$

$$C = 1 + 2(-\frac{3}{4}) = 1 - \frac{3}{2} \Rightarrow C = -\frac{1}{2}$$

$$\frac{2x-1}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1} = \frac{A(x^2+1)}{(x+1)(x^2+1)} + \frac{(Bx+C)(x+1)}{(x+1)(x^2+1)}$$

$$\frac{Ax^2+A+Bx^2+Bx+Cx+C}{(x+1)(x^2+1)} = \frac{2x-1}{(x+1)(x^2+1)}$$

$$Ax^2+Bx^2=0$$

$$A=-B$$

$$-1-C = -(2-C)$$

$$-1-C = -2+C$$

$$+2+C \quad +2+C$$

$$1=2C$$

$$\frac{1}{2}=C$$

$$Bx+Cx=2x$$

$$B+C=2$$

$$B=2-C$$

$$B=2-\frac{1}{2}$$

$$B=1\frac{1}{2}=\frac{3}{2}$$

$$A+C=-1$$

$$A=-1-C$$

$$A=-1-\frac{1}{2}=-1\frac{1}{2}=-\frac{3}{2}$$

$$\frac{2x-1}{(x+1)(x^2+1)} = \frac{-3}{2(x+1)} + \frac{\frac{3}{2}x + \frac{1}{2}}{(x^2+1)} = \frac{-3}{2(x+1)} + \frac{3x+1}{2(x^2+1)}$$

$$a^3+b^3=(a+b)(a^2-ab+b^2)$$

$$a^3-b^3=(a-b)(a^2+ab+b^2)$$